substituted into (103) with the force law of (104), we obtain

$$\text{(106)}\quad d^2S/dy^2 = [S(y-\theta) - 2S(y) + S(y+\theta)]\cdot\{1 - \alpha[S(y-\theta) - S(y+\theta)]\}\,,$$

where

$$y = T - N\theta\,.$$

Equation (106) is an uncommon type of differential-difference equation and is not readily solved, even numerically. An approximate solution can be found by expanding $S(y\pm\theta)$ in powers of θ and retaining fourth order terms. If dissipation is nonzero, however small, nearly-steady oscillations of the type suggested by Fig. 32 are obtained. The frequencies of oscillation predicted by the approximate theory are, however, quite different from those obtained from numerical integration of the transient equations. It thus appears that this picture of the permanent regime in the lattice is qualitatively correct, but that eq. (106) must be solved if parameters of the permanent regime are to be calculated [21].

One interesting application of these results is to the von Neumann-Richtmyer integration of the flow equations. If artificial viscosity is too small, the results of such an integration oscillate wildly. This has sometimes been interpreted as instability of the numerical integration procedure; it is in fact the true physical behavior of the lumped-constant system used to model the continuum for purposes of numerical integration.

REFERENCES

[1] R. Courant and K. O. Friedrichs: *Supersonic Flow and Shock Waves* (New York, 1948). This contains much material basic to Sects. **1-3**.

[2] G. E. Duvall: *Les ondes de détonation* (Paris, 1962), p. 337.

[3] D. R. Bland: *Journ. Inst. Math. Appl.*, **1**, 56 (1965).

[4] J. O. Erkman and G. E. Duvall: *Developments in Mechanics*, edited by T. C. Huang and M. W. Johnson Jr., vol. **3**, Pt. 2 (New York, 1965), p. 179.

[5] W. Herrmann: *Wave Propagation in Solids* (New York, 129), p. 129.

[6] G. E. Duvall and Y. Horie: *Proc. Fourth Symposium on Detonation, Oct. 12-15, 1965* (Washington, D. C., 1966), p. 248.

[7] R. D. Richtmyer and K. W. Morton: *Difference Methods for Initial-Value Problems*, 2nd Ed. (New York, 1967).

[8] T. J. Ahrens and G. E. Duvall: *Journ. Geophys. Res.*, **71**, No. 18, 4349 (1966).

[9] Y. Horie and G. E. Duvall: *Proc. Army Symposium on Solid Mechanics, 1968*, p. 127; available from Applied Mechanics and Research Laboratory, Army Materials and Mechanics Research Center, Watertown, Mass.

[10] B. R. Breed, C. L. Mader and D. Venable: *Journ. Appl. Phys.*, **38**, 3271 (1967).
[11] B. M. Butcher, L. M. Barker, D. E. Munson and C. D. Lundergan: *Am. Inst. Aeronautics and Astronautics Journ.*, **2**, 977 (1964).
[12] F. W. Tuler: Sandia Laboratories, Albuquerque, N. Mex., private communication.
[13] F. A. McClintock: *International Journ. Fract. Mech.*, **4**, 101 (1968).
[14] J. S. Rinehart and J. Pearson: *Behavior of Metals Under Impulsive Loads* (New York, 1954).
[15] G. R. Fowles and G. D. Anderson: Poulter Laboratories Internal Report 032-59, Stanford Research Institute, Menlo Park, Calif. (1959).
[16] W. E. Drummond: *Comments on the cutting of metal plates with high explosive charges*, paper no. 57-A-89, American Society of Mechanical Engineers, 345 E. 47th St., New York, N. Y. 10017 (1957).
[17] J. O. Erkman: *Journ. Appl. Phys.*, **32**, 939 (1961).
[18] J. Lethaby and I. C. Skidmore: S.W.A. Branch Note No. 3/59, AWRE, Aldermaston, Berks. (March 1959).
[19] S. J. Tupper: *On the propagation of plane stress waves generated in a thick steel plate by a surface explosion*, A.R.D.E. Report (B) 12/61 (Sept. 1961).
[20] R. Manvi, G. E. Duvall and S. C. Lowell: *Int. Journ. Mech. Sci.*, **11**, 1 (1969).
[21] G. E. Duvall, R. Manvi and S. C. Lowell: *Journ. Appl. Phys.*, **40**, 3771 (1969).